

Prediction of the Poisson's ratio of porous materials

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An equation is presented for the prediction of the Poisson's ratio of porous materials. The equation is strictly derived for spherical porosity and isotropic materials and it is valid for the whole porosity range. For low porosity, the equation coincides with a published equation, which has been verified in the past by comparison with extensive experimental data. For the high-porosity range, the theoretical variation of the Poisson's ratio exhibits a trend converging to a value $\nu_p = 0.5$, when the porosity increases to $P = 1$. A similar converging trend has been found in other theoretical studies, but a rigorous experimental verification of such variations has still to be carried out.

1. Introduction

Although the elastic behaviour of porous materials has received extensive experimental and theoretical treatment, there is very little work dealing directly with the porosity dependence of Poisson's ratio

$$\nu = \frac{\Delta q/q}{\Delta l/l} \quad (1)$$

where q is the cross-section perpendicular to the stress direction, Δq the areal contraction under stress, l the length of the specimen in the stress direction, and Δl the elongation in the stress direction.

Because the overall volume of a specimen under stress is either enhanced or remains at least constant ($\Delta V/V \geq 0$) it was deduced [1] that the range of variation of Poisson's ratio is $0 < \nu \leq 0.5$. Formal thermodynamic considerations, however, result in a range of variation for the Poisson's ratio of isotropic materials of $-1 \leq \nu < 0.5$ [2]. Nevertheless, the available experimental data on the Poisson's ratio of porous materials, as considered here, lie in the range $0 < \nu \leq 0.5$. Being a dimensionless parameter, Poisson's ratio is a very useful elastic property because it enters in a number of equations describing the fracture and deformation behaviour of materials. Therefore, it is theoretically interesting and practically useful to obtain an accurate assessment of its dependence on porosity.

It has frequently been considered that the variation of Poisson's ratio with porosity is negligible [3], especially at high porosities, although this behaviour was not always confirmed by experimental trends. Indeed, most of the proposed theoretical equations have not

been compared extensively with experimental data [4–8] and, for example, two-dimensional finite element methods (FEM) of limited validity had to be used for comparison purposes [7]. It is possible to estimate Poisson's ratio using any two of the bulk, shear or Young's moduli, but an accurate estimation of the effective Poisson's ratio is difficult because of the large error involved. This is why many experimental investigations report only the qualitative trend of the variation of Poisson's ratio with porosity [9].

A quantitative effort was made by applying the self-consistent oblate spheroidal theory to fit experimental elastic constant data in porous ceramics [10]. The approach, however, as the author pointed out, seems to overestimate the value of Poisson's ratio for porosities above 30%.

Recently, a new equation has been proposed and compared extensively with experimental data available on porous ceramics [11]. This equation can be written as

$$\nu_P = 0.5 - \frac{(1 - P^{2/3})^{1.21} [2(3 - 5P)(1 - 2\nu_0) + 3P(1 + \nu_0)]}{4(3 - 5P)(1 - P)} \quad (2)$$

where P is the volume fraction of porosity and ν_0 is the Poisson's ratio of the porous-free material.

Although the equation is strictly valid for spherical geometry, very good agreement with the experimental data was found, indicating a minor effect of the pore shape on the Poisson's ratio–porosity dependence. Moreover, it was shown that the equation predicts

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the experimental data better than other proposed theories [4–6, 12] in its range of validity [13]. The disadvantage of Equation 2 is its validity only for low concentrations of pores ($P < 0.5$) and therefore it is unsuitable for considering foam-like or cellular materials. This is why in this paper an approach is presented for the dependence of the Poisson's ratio of porous materials valid for the whole range of porosity, based on published equations for the Young's and bulk moduli and the relation between elastic constants in isotropic materials.

2. Derivation of the equation and discussion

2.1. Young's modulus

An expression for the Young's modulus of porous materials has been derived by substituting the real pore structure by a model microstructure of spheroids having the same surface to volume ratio as the real pores [14]. Two factors describing the pore structure remain relevant for the calculation [15]:

- (i) the shape factor (z/x), defined by the axial ratio of the substitutional spheroids; and
- (ii) the orientation factor, defined as $\cos^2\alpha$, where α is the angle between the stress direction and the rotational axis of the substitutional spheroid.

These two parameters can be accurately determined by quantitative image analysis of polished surfaces and stereological relations [15] making the model very useful for practical applications. The final expression for the Young's modulus of the porous material can be written as [14, 16]

$$E_p = E_0(1 - P^{2/3})^{1.21\Gamma}$$

with

$$\Gamma = \left(\frac{z}{x}\right)^{1/3} \left\{ 1 + \left[\left(\frac{z}{x}\right)^{-2} - 1 \right] \cos^2\alpha \right\}^{1/2} \quad (3)$$

where E_0 is the Young's modulus of the fully dense material. For the special case of spherical pores ($z/x = 1$), the Young's modulus–porosity dependence can be expressed as

$$E_p = E_0(1 - P^{2/3})^{1.21} \quad (4)$$

These equations have been extensively tested by comparison with experimental values on a variety of porous materials containing different pore structures and low porosities ($P < 0.5$) [16, 17]. Excellent agreement has been found between theory and experiment. For the high-porosity range, however, little experimental work has been reviewed until now for comparison purposes. As a first example for this comparison, Fig. 1 shows calculated and experimental values for highly porous materials such as cellular alumina [18], polymer foam [19] and porous silica gel [20]. Because no information about pore shape and orientation was available from the experimental studies, definite values ($z/x = 4.5$; $\cos^2\alpha = 0.33$) for the shape factor and the orientation factor have been used for all calculations, which refer to prolate isotropically oriented

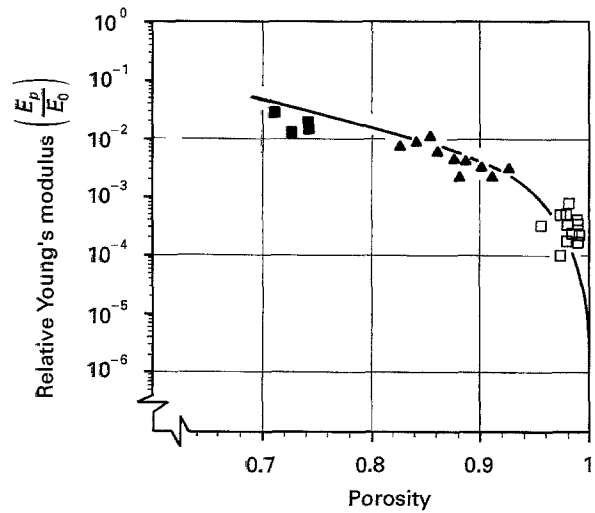


Figure 1 Variation of the relative Young's modulus with porosity for highly porous materials: (\blacktriangle) cellular alumina [18], (\square) polymer foam [19], (\blacksquare) porous silica gel [20], Equation 3 with $z/x = 4.5$, (—) $\cos^2\alpha = 0.33$.

porosity and lead to the best fit. Although the agreement between experiment and theory as shown in Fig. 1 confirms the applicability of Equation 3 for the high-porosity range, more experimental data are needed for a definite verification of the equations for highly porous or cellular materials. Model highly porous glasses with well-defined porous structures, prepared by sintering of hollow microspheres in a glass matrix, can be conveniently used for this purpose [21].

2.2. Bulk modulus

Although an equation that gives the variation of the bulk modulus with the volume fraction of pores and the pore structure (shape and orientation of pores) has been published [22, 23], its mathematical complication makes it unuseful for practical purposes, except its special case for spherical porosity.

For low concentration of spherical pores, the equation for the porosity dependence of the bulk modulus can be written as [22, 23]

$$K_{P,1} = K_0 \frac{2(1 - 2\nu_0)(3 - 5P)(1 - P)}{2(3 - 5P)(1 - 2\nu_0) + 3P(1 + \nu_0)} \quad (5)$$

For the high-porosity range, an expression has been derived for the compressibility of a foam-like glass considering the stresses at equilibrium just before it "solidifies" [24]. According to this, the porosity dependence of the bulk modulus for spherical pores and high porosity can be written as

$$K_{P,2} = K_0 \frac{2(1 - 2\nu_0)(1 - P)}{3(1 - \nu_0)} \quad (6)$$

In order to obtain a single equation for the whole porosity range, Equations 5 and 6 may be mathematically joined by means of the following function

$$s = \frac{1}{1 + e^{-a(P-b)}} \quad (7)$$

where the fitting factors ($a = 100$; $b = 0.4$) have been chosen for the smoothest joint. Thus, the new

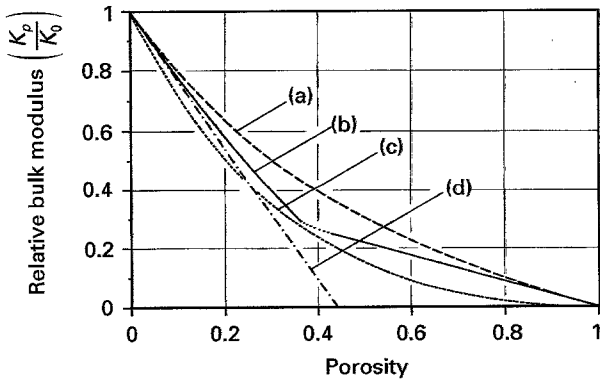


Figure 2 Variation of the relative bulk modulus with porosity: (a) Kerner [26], (b) Equation 8, (c) Ramakrishnan-Arunachalam [7], (d) Okana [25].

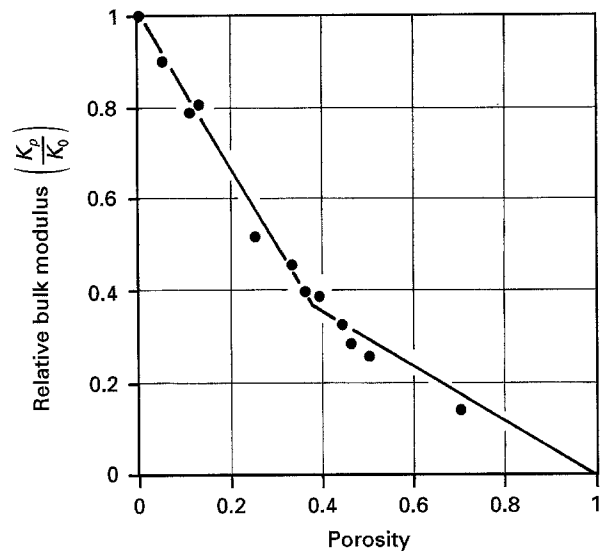


Figure 3 Variation of the relative bulk modulus of porous glass: (●) experimental values [24], (—) calculated values, Equation 8.

equation for the porosity dependence of the bulk modulus, valid for the whole range of porosity can be written as

$$K_P = (1 - s)K_{P,1} + sK_{P,2} \quad (8)$$

Fig. 2 shows the theoretical curve according to Equation 8 for Poisson's ratio $\nu_0 = 0.2$. Also shown in the figure are the predictions of other theoretical models [7, 25, 26]. Two of them appear to be upper [26] and lower [7] bounds for Equation 8 presented here, one model [24] predicts vanishing bulk modulus for a porosity lower than $P = 1$.

Former experimental data for porous glass containing a wide range of porosity [24] can be used for confirmation of Equation 8. Fig. 3 shows the comparison between the calculated and experimental values confirming good agreement between theory and experiment.

2.3. Poisson's ratio

The relation between Young's modulus, bulk modulus and Poisson's ratio for isotropic materials can be

written as

$$\nu = 0.5 - \frac{E}{6K} \quad (9)$$

Because Equations 5 and 6 (and hence Equation 8) are valid for spherical porosity, the following considerations for the porosity dependence of the Poisson's ratio will be restricted to this special case. Replacing the moduli (E and K) in Equation 9 by their porosity functions according to Equations 4 and 8, the porosity function of the Poisson's ratio of isotropic porous materials with spherical pores is obtained

$$\nu_P = 0.5 - (1 - P^{2/3})^{1.21}$$

$$\left[4 \left[(1 - s) \frac{(3 - 5P)(1 - P)}{2(3 - 5P)(1 - 2\nu_0) + 3P(1 + \nu_0)} + s \frac{(1 - P)}{3(1 - \nu_0)} \right] \right] \quad (10)$$

with

$$s = \frac{1}{1 + e^{-100(P-0.4)}}$$

This equation represents a better solution for the dependence of Poisson's ratio on porosity than the equation proposed earlier [11], because (a) it is valid for the whole porosity range, and (b) it provides a plausible result for the borderline case of a material without pores ($P = 0 \rightarrow \nu_P = \nu_0$; note that $P = 0$ results in $s \rightarrow 0$, Equation 7).

The low-porosity range of Equation 10 coincides with Equation 2, which has been experimentally confirmed by comprehensive comparison of calculated and measured values [11, 13] and by comparison with other theories [13]. Thus the experimental verification of Equation 10 for low porosity will not be repeated here, although it remains an open question, why the slope of the curve possesses a maximum, or, in other terms, why the slope is as it is. Interpretation and re-examination is desired here. Fig. 4 shows the theoretical variation of the Poisson's ratio with porosity for different values of the Poisson's ratio of the bulk material ($\nu_0 = 0.1, 0.25, 0.45$) as calculated from Equation 10. The slope of the curves itself again calls for

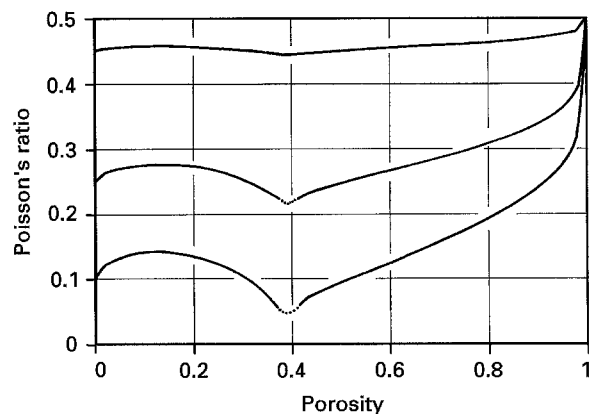


Figure 4 Theoretical variation of the Poisson's ratio with porosity, Equation 10, for different values of ν_0 (0.1, 0.25, 0.45).

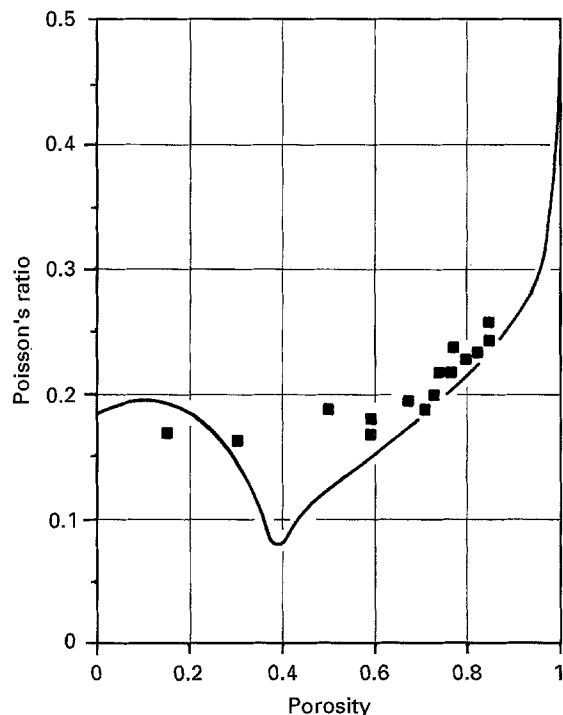


Figure 5 Variation of the Poisson's ratio of porous gel-derived silica with porosity: (■) experimental values [27], (—) calculated values, Equation 10.

reconsideration and further examination. The theoretical variation of the Poisson's ratio with porosity exhibits a general trend of converging to a particular value, $v_p = 0.5$, when total porosity is approached – meaning the elastic moduli become zero. Similar trends have been found by other authors, however, with other convergence values (Zimmerman's equation $v_p = 0.2$ for $P = 1$ [8]; Ramakrishnan–Arunachalam's equation $v_p = 0.25$ for $P = 1$ [7]). Because of the lack of sufficient and reliable experimental data, a rigorous experimental verification of the converging trend of the Poisson's ratio–porosity dependence for the high-porosity range also remains an open question if, in addition, Fig. 5, showing data for a gel-derived porous silica [27], encourages us to follow the line proposed by Equation 10. This is a challenge, not a statement.

Conclusion

An equation has been derived for calculating the Poisson's ratio of porous materials containing spherical porosity. The shape of the calculated Poisson's ratio versus porosity curves showing a relative maximum at low porosities and a minimum at porosities $P \approx 0.4$

needs further re-examination and clarification. For low porosities, the equation coincides with a previously published formula, which has been verified in the past by comparison with extensive experimental data. For high porosities, a converging trend of the Poisson's ratio to a value $v = 0.5$ for $P = 1$ was found. Although similar trends have been predicted by other theories, there has been no rigorous experimental verification of this behaviour due to the lack of available reliable data.

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